ERTH 455 / GEOP 555
Geodetic Methods

– Lecture 23: Modeling - Plate Kinematics –

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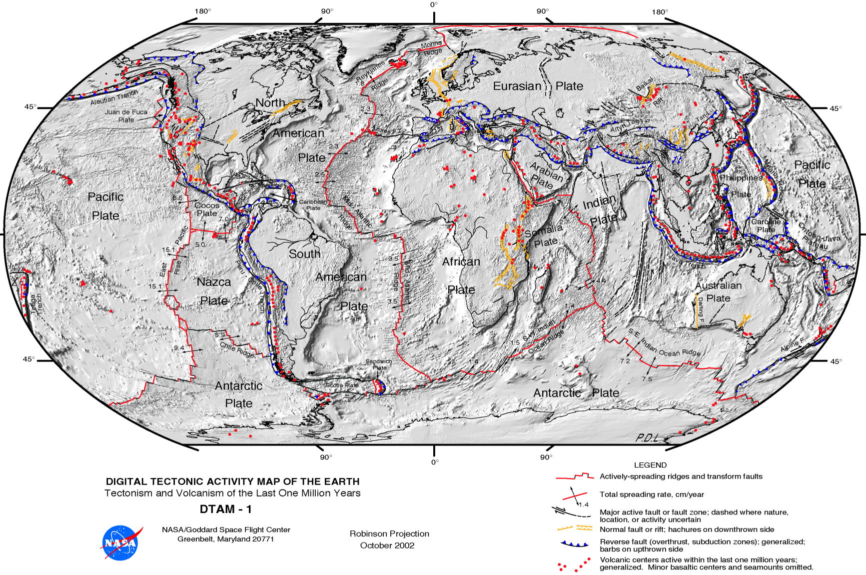
Tectonic Activity



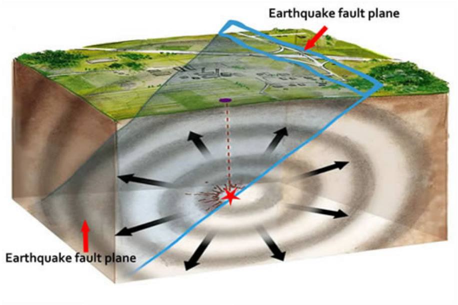
NASA

Plates and Boundaries

- plates are rigid, relative motions occur on their boundaries
- how many plates / microplates are there?
- plate boundaries have some finite width: plate boundary zones
  - can be narrow: $< 10\,\text{km}$
  - or very wide: $500 - 1000\,\text{km}$
- relative motion occurs on faults, or breaks in the Earth's lithosphere

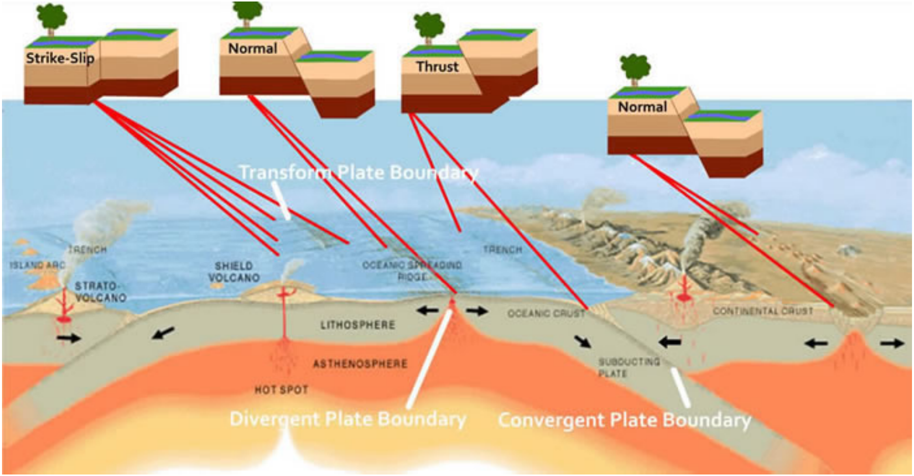
Faults

- Faults are (approximately) planar surfaces
- Motion on either side of the surface relative to the other
- Direction of motion is slip direction
- Motion driven by plate tectonics
- Nature of slip depends on depth:
  - shallow: fault stuck together (friction), slip occurs suddenly in earthquakes
  - deep: fault slips mostly at steady rate



Valerie Thomas, USGS & Anthony Guarino, Caltech

Types of Faults



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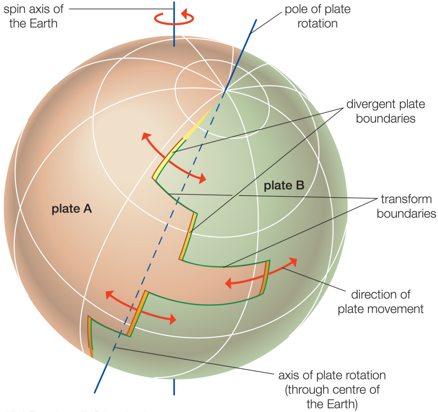
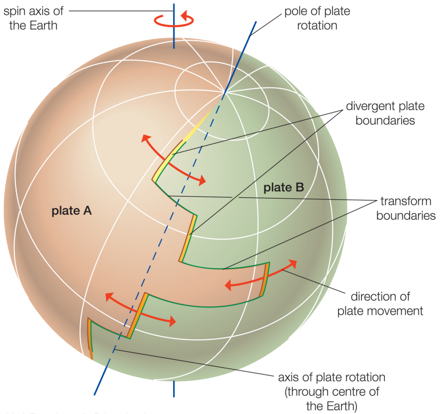
Motion on a sphere



- rigid motion on sphere is about geometric axis
- 2 equivalent ways to describe:
  - pole of rotation (Euler Pole) and angular speed (deg/Myr)
  - angular velocity vector

Motion on a sphere

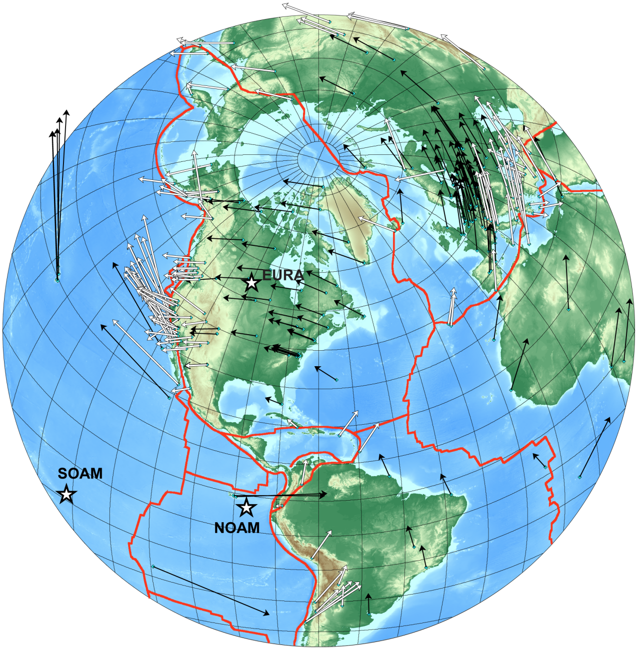


- relative plate motion direction given by transform fault direction
- transform fault indicates small circle about rotation pole
- relative plate motion rate given by seafloor magnetic isochrons
- GPS velocities are direct measure of plate motion direction and magnitude
- GPS velocities are normal to great circle passing through pole of rotation

Geologic Plate Motion Models

- Relative plate motion models based on a combination of
  - Mid-ocean ridge spreading rate (from marine magnetic anomalies)
  - transform fault azimuths
  - earthquake slip vectors (hard)
- some plates have little or no data (Caribbean, Philippine Sea Plates)
- Common models: NUVEL-1, revised to NUVEL-1A
- New model: MORVEL (DeMets et al, 2010)

Motion on a sphere



courtesy: Jeff Freymueller, UAF

Conversion between Euler Parameters and Rotation Vector

- Rotation Vector: $\Omega(\omega_x, \omega_y, \omega_z)$
- Euler Parameters: latitude λ, longitude ϕ, angular speed s
- Euler to Rotation:

$$\begin{aligned}
\omega_x &= s\,cos(\lambda)\,cos(\phi) \\
\omega_y &= s\,cos(\lambda)\,sin(\phi) \\
\omega_z &= s\,sin(\lambda)
\end{aligned}$$

- Rotation to Euler:

$$\begin{aligned}
\lambda &= arctan\left(\frac{\omega_z}{\sqrt{\omega_x^2 + \omega_y^2}}\right) \\
\phi &= arctan\left(\frac{\omega_y}{\omega_x}\right) \\
s &= \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}
\end{aligned}$$

Calculating Site Velocities

- Easiest to use angular velocity vector to compute site velocities
- Cross product of site location P (ECEF: P = [X,Y,Z]) with plate angular velocity:

$$\vec{v} = \vec{\omega} \times \vec{P}$$

- expand cross product to rewrite as matrix equation:

$$\begin{aligned} \vec{v} &= \vec{\omega} \times \vec{P} \\ &= (Z\omega_y - Y\omega_z)\hat{x} + (X\omega_z - Z\omega_x)\hat{y} + (Y\omega_x - X\omega_y)\hat{z} \end{aligned}$$

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- want m/yr:
  - time-factor: 10^{-6}
  - degrees-to-radians-to-arclength: $\frac{\pi}{180}R$
  - where R is mean Earth radius: $R = 6,378,137\,\mathrm{m}$

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- need location conversion from geodetic to geocentric frame (lat,lon to X,Y,Z)
- need velocity conversion from geocentric to local frame (X,Y,Z to N,E,U)

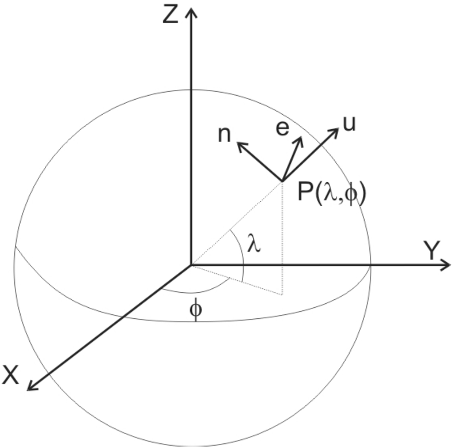
Recall: Geodetic to Geocentric

$$
\begin{aligned}
X &= (N+h)cos(\lambda)cos(\phi) \\
Y &= (N+h)cos(\lambda)sin(\phi) \\
Z &= \left(\frac{a^2}{b^2}N+h\right)sin(\lambda)
\end{aligned}
\qquad
\begin{aligned}
N &= \frac{a^2}{\sqrt{a^2cos^2\lambda+b^2sin^2(\lambda)}} \\
b &= -fa+a
\end{aligned}
$$

- *a*: semi-major axis of ellipsoid (WGS84: 6378137.0 m)
- *f*: flattening of ellipsoid (WGS84: 1/298.257223563)
- *b*: semi-minor axis of ellipsoid
- λ, ϕ, h: geodetic latitude, longitude, height (above ellipsoid)
- X, Y, Z: ECEF Cartesian coordinates

Recall: Geocentric to Local

- origin of this datum is point of your choice on the surface of the Earth
- right handed coordinate system:
  - *U* is vertical. i.e., perpendicular to local equipotential surface, points up
  - *N* is in local horizontal plane and points to geographic north
  - *E* is in local horizontal plane and points to geographic east
- as ECEF (XYZ) units are meters, local units are meters, too

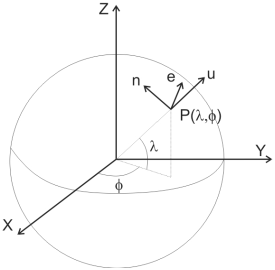


Recall: Geocentric to Local

Combine 3 rotations to align geocentric with NEU frame:

$$\begin{bmatrix} v_N \\ v_E \\ v_U \end{bmatrix} = \begin{bmatrix} -sin(\lambda)cos(\phi) & -sin(\lambda)sin(\phi) & cos(\lambda) \\ -sin\phi) & cos(\phi) & 0 \\ cos(\lambda)cos(\phi) & cos(\lambda)sin(\phi) & sin(\lambda) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$\vec{V}_{local} = R\vec{V}_{ecef}$$



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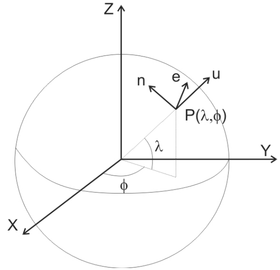
$$\vec{V}_{local} = R\vec{V}_{ecef}$$

- inverse matrix of R can be used to convert local to ECEF
- since R is rotation matrix:

$$R^{-1} = R^T$$

- Therefore:

$$\vec{V}_{ecef} = R^T\vec{V}_{local}$$



Estimating Plate Angular Velocity

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Estimating Plate Angular Velocity

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$$d = Gm$$

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- ...
- Invert for m:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = (G^T W G)^{-1} G^T W \vec{v}$$

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- Invert for m:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = (G^T W G)^{-1} G^T W \vec{v}$$

- where the weight matrix W is the inverse of the velocity covariance matrix C_V (comes from processing)
- model covariance matrix is $C_\omega = (G^T C_V^{-1} G)^{-1}$

Estimating Plate Angular Velocity

- How many site velocities do you need?

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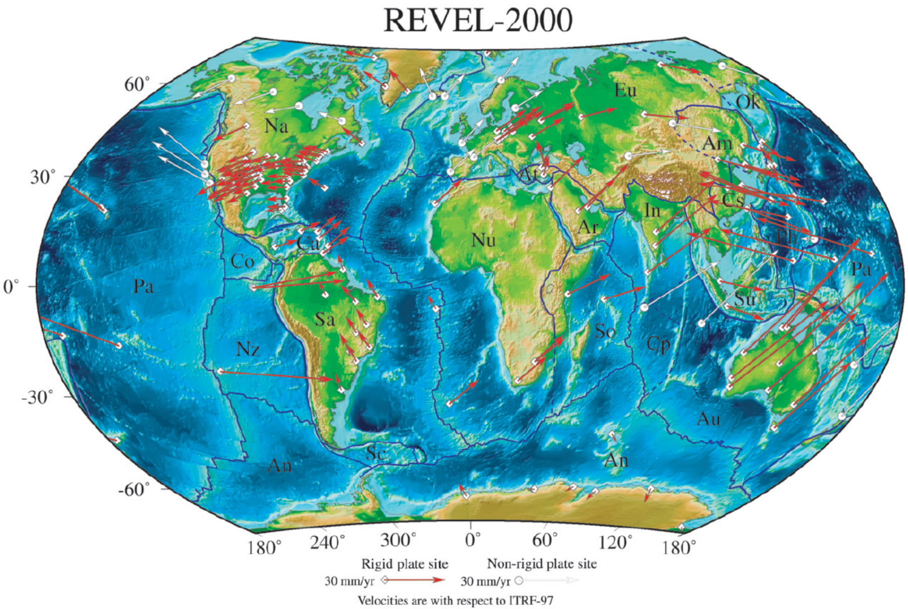
- 3 data in each site velocity (v_N, v_E, v_U)
- But: plate model predicts no vertical - only horizontals count!

Estimating Plate Angular Velocity

- How many site velocities do you need?
- 3 parameters in plate angular velocity vector
- 3 data in each site velocity (v_N, v_E, v_U)
- But: plate model predicts no vertical - only horizontals count!
- Need velocities for at least 2 sites to constrain plate angular velocity
- The more GPS velocities and the farther apart, the better determined is plate angular velocity

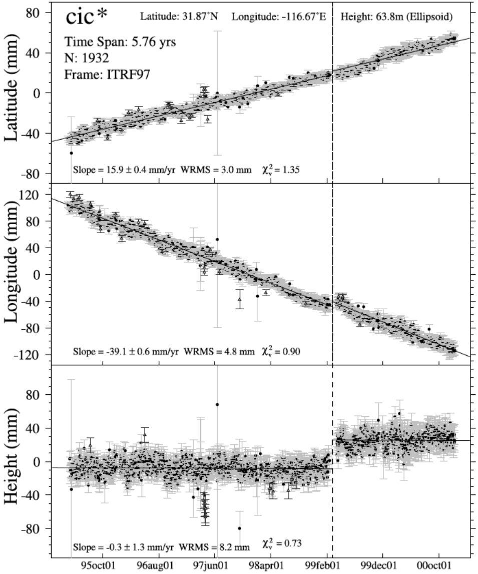
Example: REVEL-2000



Example: REVEL-2000

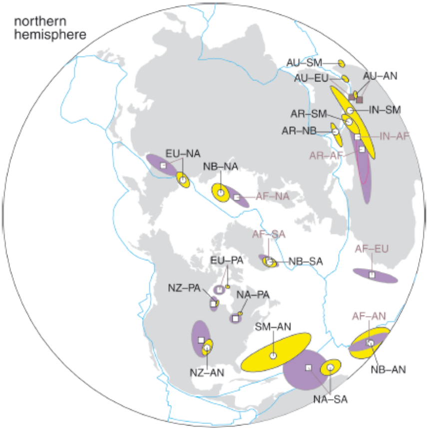
- REVEL = “recent velocities”
- Global plate motion model based entirely on GPS data for 19 plates
- Data from 1993 though 2000.
- combination of many continuous sites and repeat campaign survey data
- first model with essentially global coverage
- 2/3 of tested plate pairs agree within uncertainties with NUVEL-1A (geologic 3Myr average)

Example: REVEL-2000



- example of data used in model
- long time series in ITRF97
- PPP solutions
- fit linear trends plus offset (here: combination of co-located sites)
- outlier rejection, quality control
- 345,000 station days

Example: GEODVEL



Argus et al., 2010, GJI

Rotation poles and confidence ellipses for adjacent plate pairs for GEODVEL (open circles, yellow) and NUVEL-1A (open squares, violet)

Example: GEODVEL

- GEODVEL = “GEODesy VELocity”
- based on GPS, VLBI, SLR, DORIS in ITRF2005
- relative angular velocities for 11 major plates
- also provides absolute plate poles

Example: GEODVEL

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Plate Fixed Reference Frames

- "Velocities with respect to PLATE-NAME"
- very convenient for visualization purposes and modeling of tectonic deformation
- To convert into plate-fixed frame we need plate motion and velocities in the same geodetic frame (e.g., ITRF2008)
- Transformation:

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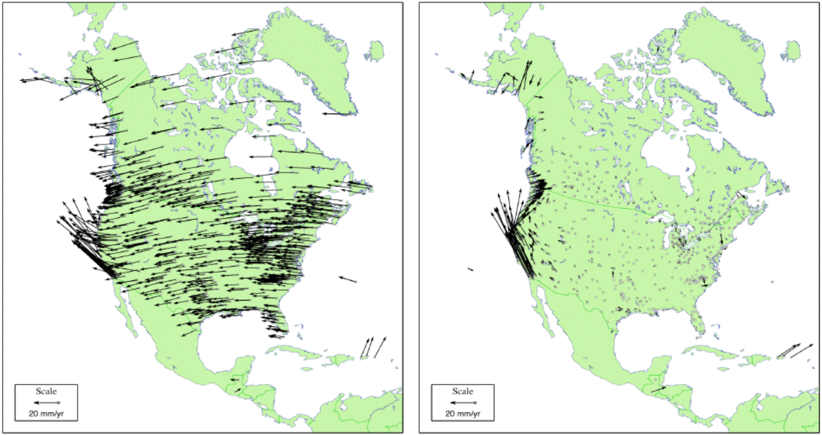
- very convenient for visualization purposes and modeling of tectonic deformation
- To convert into plate-fixed frame we need plate motion and velocities in the same geodetic frame (e.g., ITRF2008)
- Transformation: subtract predicted motion based on plate angular velocity from observed velocity

Reference Frames – ITRF vs. fixed (stable North America)

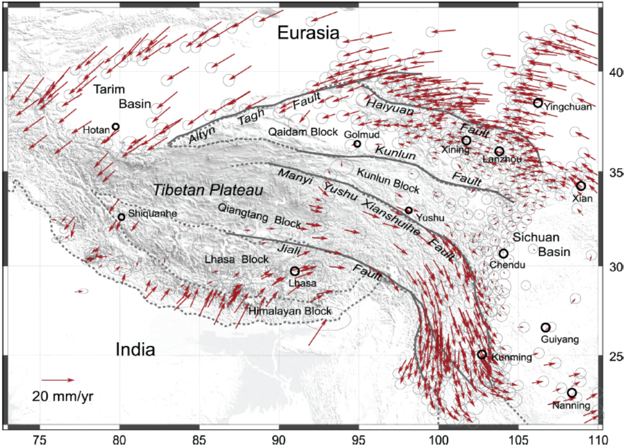


courtesy: Jeff Freymueller, UAF

Reference Frames – stable North America

- extension across Basin and Range
- Shear on San Andreas System
- Subduction strain in Cascadia, Alaska
- et al.

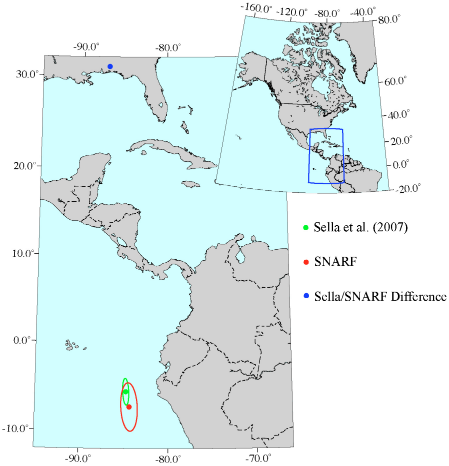
"Tibetan Plateau Reference Frame"



Gan et al. (2007) explained these motions in terms of a series of blocks separated by mostly strike-slip faults ➔ plateau is deforming, but not changing area.

courtesy: Jeff Freymueller, UAF

NOAM Poles



courtesy: Jeff Freymueller, UAF

- past studies: common that NOAM poles not within each others' confidence ellipses
- Difference between SNARF and Sella et al. (2007) is rotation about pole in SE US.

Why is NOAM Pole poorly determined?

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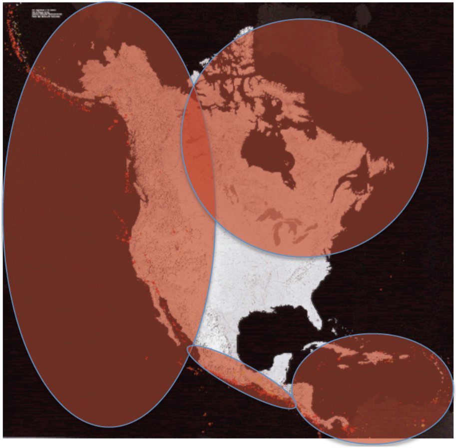

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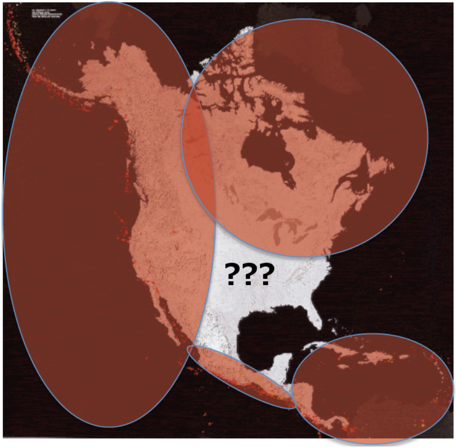

courtesy: Jeff Freymueller, UAF

- tectonics in western North America
- glacial isostatic adjustment in northern North America
- SE is thought to be stable on geologic and geodetic time scales
- limited area to determine plate angular velocity, susceptible to bias