ERTH 455 / GEOP 555
Geodetic Methods

– Lecture 25: Modeling - Slip Inversion –

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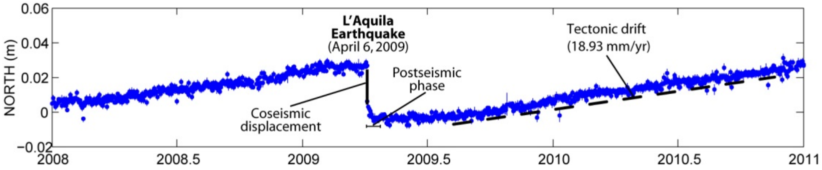
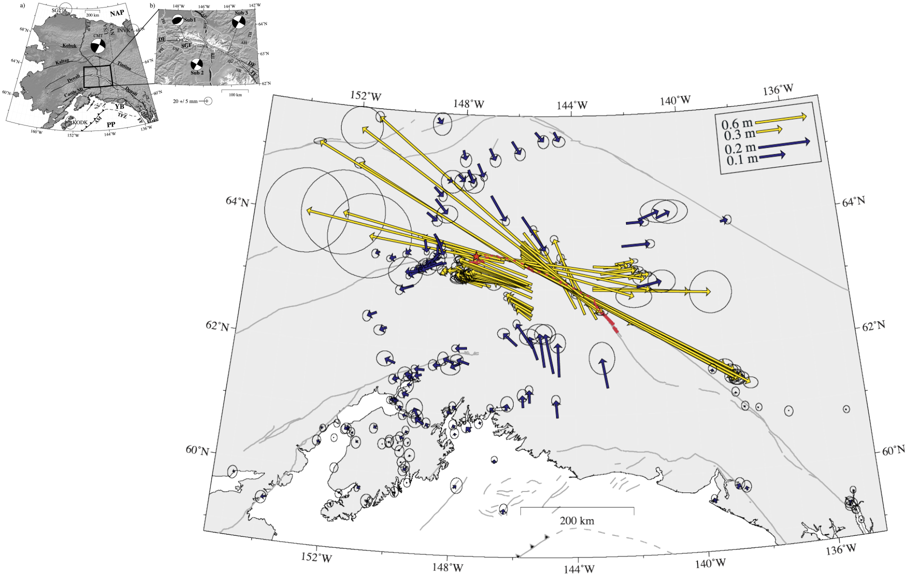
The deformation cycle



Roberto Devoti, INGV

- Earthquake: sudden slip on fault
- M_w 4-5: a few centimeters average slip on fault
- M_w 7: a few meters average slip on fault
- M_w 9: 10-20+ meters average slip on fault
- L'Aquila earthquake: M_w 5.9 - displacements depend on distance, magnitude, fault geometry

Co-Seismic: The 2002 M_w=7.9 Denali Earthquake



Hreinsdóttir et al., JGR, 2006

Co-Seismic: The 2002 M_w=7.9 Denali Earthquake

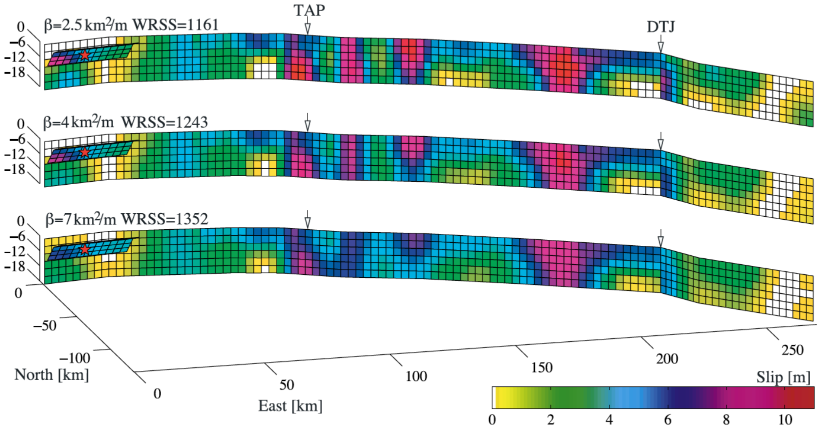


Figure 10. Range of reasonable coseismic slip models from the roughest (β = 2.5 km^2/m) to the smoothest (β = 7 km^2/m). The axes show easting, northing, and depth in km. TAP, Trans-Alaska pipeline; DTJ, Denali-Totschunda fault junction. Red star indicates the Denali Fault earthquake epicenter.

Hreinsdóttir et al., JGR, 2006

The deformation Cycle: Post-seismic

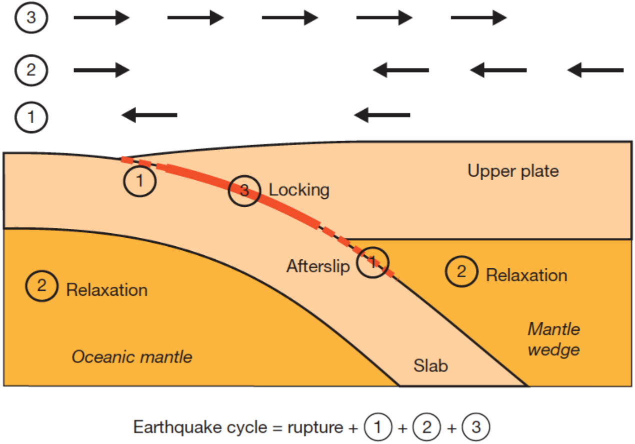


Figure 2 | Three primary processes after a subduction earthquake. (1) Aseismic afterslip occurs mostly around the rupture zone, (2) the coseismically stressed mantle undergoes viscoelastic relaxation, and (3) the fault is relocked. Arrows at the top show the sense of horizontal motion of Earth's surface, relative to distant parts of the upper plate, caused by each of these three processes.

Wang et al., 2012, Nature

The deformation Cycle: Post-seismic

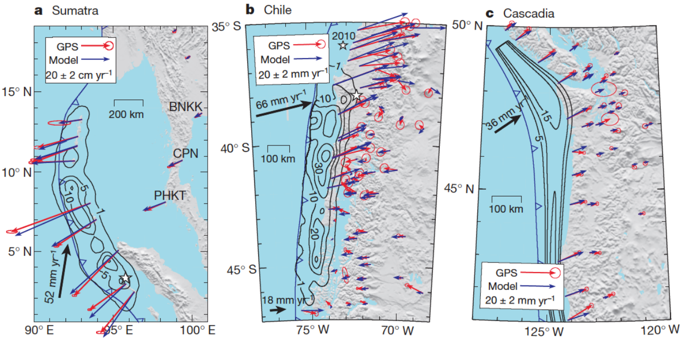
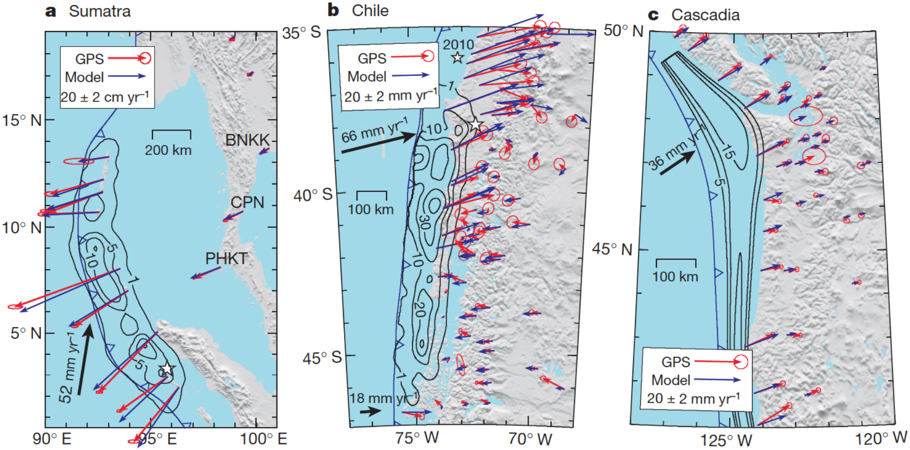


Figure 3 | GPS- (red) and model-predicted (blue) surface velocities for three subduction zones that are at different stages of the earthquake cycle. a, At Sumatra, one year after the $M_w = 9.2$ earthquake of 2004 (refs 20 and 21) (epicentre shown by star), all sites move seaward. Shown are ~1-year average GPS velocities. More recent data show the same pattern[22]. Coseismic fault slip (contoured in metres) is based on ref. 56. Longer (~3-years) time series from the three labelled far-field sites (BNKK, CPN, PHKT)[32] helped constrain afterslip and transient rheology (ref. 48). **b**, At Chile, four decades after the $M_w = 9.5$ earthquake of 1960, coastal and inland sites show opposing motion. Coseismic slip is from ref. 14. For sources of GPS data, see ref. 17. The northernmost areas show wholesale landward motion before the 2010 $M_w = 8.8$ Maule earthquake. **c**, At Cascadia, three centuries after the $M_w \approx 9$ earthquake of 1700, all sites move landward. The model is an updated version of ref. 8. A more comprehensive GPS compilation shows a similar deformation pattern[16].

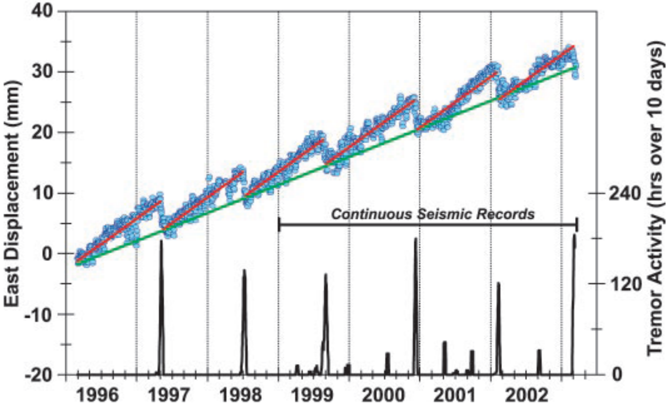
Wang et al., 2012, Nature

The deformation Cycle: Post-seismic



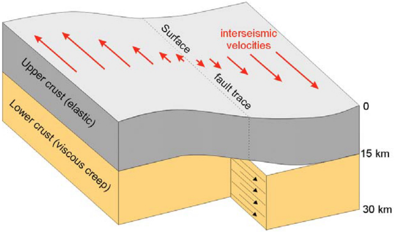
Wang et al., 2012, Nature

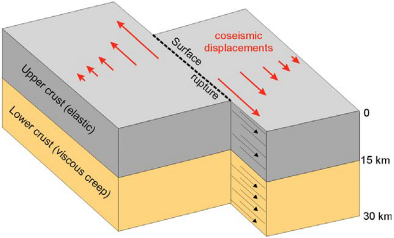
The deformation Cycle – Slow Slip



Rogers & Dragert 2003, Science

Physics of Faults





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- stick-slip sliding (seismic)
  - 2 sides of interface stuck together: friction
  - slip occurs when friction is overcome
  - slip controlled by dynamic friction, healing
- stable sliding (aseismic):
  - 2 sides slide continuously past each other
  - slip occurs all the time
  - slip controlled by plastic, ductile or viscous yielding
- transient slip also occurs (slow slip events)

Geodetic data → Slip on a Fault

How to get this?

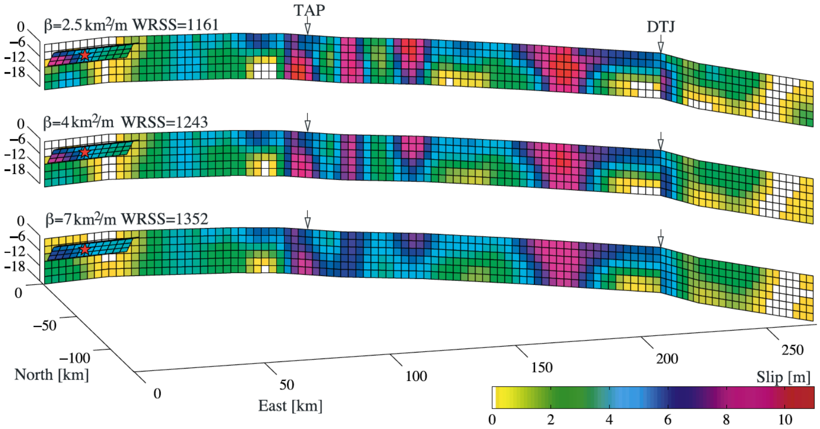

Figure 10. Range of reasonable coseismic slip models from the roughest ($\beta = 2.5\ km^2/m$) to the smoothest ($\beta = 7\ km^2/m$). The axes show easting, northing, and depth in km. TAP, Trans-Alaska pipeline; DTJ, Denali-Totschunda fault junction. Red star indicates the Denali Fault earthquake epicenter.

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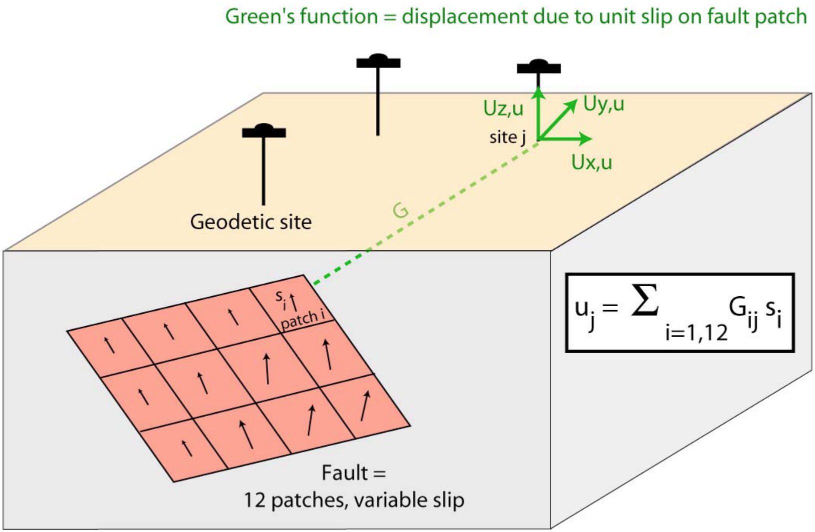

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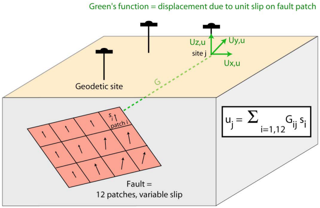
Slip on a Fault

- We want to use displacements to determine where on the fault how much slip occurred
- Ideally, we also want to know where the fault is.
- The problem is non-linear in fault geometry, but linear in slip

Green's Functions

- Is basically an impulse unit response
- Represents Earth structure ("effect of propagation from source to receiver")



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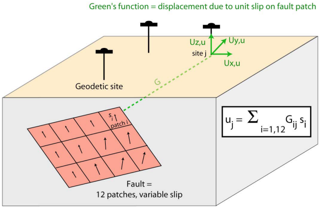
Green’s Functions

- Is basically an impulse unit response
- Represents Earth structure (“effect of propagation from source to receiver”)
- Think *“Given this Earth structureHow much displacement will I get here when the fault over there slips 1 unit (e.g. 1 m)”*
- Due to linearity you can scale this with different amounts of slip, say 25 m or 33 cm which results in scaled displacement



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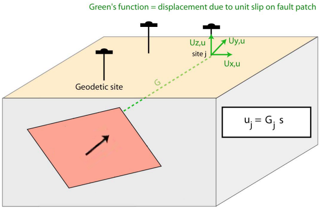
Green's Functions

- Simple earthquake: 1 fault surface with uniform strike dip, rake, slip
- Displacement at a location can be written as unit slip on that geometry times amount of slip



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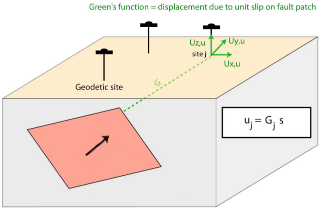
Green's Functions

- Simple earthquake: 1 fault surface with uniform strike dip, rake, slip
- Displacement at a location can be written as unit slip on that geometry times amount of slip

$$u = G * s$$

  - u is data vector
  - s is model vector
  - G is design matrix made of Green's functions
- G can be analytical expressions of derived from numerical models



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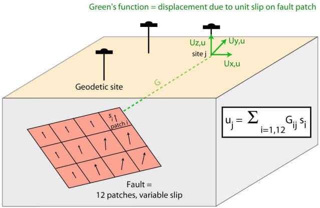
Green's Functions

- Complex earthquake: non-uniform strike dip, rake, slip
- complex fault geometry
- displacement at given site is sum of contributions of N fault patches

$$u_j = \sum_{i}^{N} G_{ij} * s_i$$



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Green's functions

Which primary directions of slip can we distinguish?

Green's functions

Which primary directions of slip can we distinguish?

- Strike-Slip (ss), Dip-Slip (ds), Opening (op)
- usually separated into their own Green's functions:

$$u_j = \sum_{i=1}^{N} \left[G_{ij}^{ss} s_i^{ss} + G_{ij}^{ds} s_i^{ds} + G_{ij}^{op} s_i^{op} \right]$$

Green's functions

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$$u_j = \sum_{i=1}^{N} \left[G_{ij}^{ss} s_i^{ss} + G_{ij}^{ds} s_i^{ds} + G_{ij}^{op} s_i^{op} \right]$$

- further separated into 3 displacement components:

$$u_j, x = \sum_{i=1}^{N} \left[G_{ij,x}^{ss} s_i^{ss} + G_{ij,x}^{ds} s_i^{ds} + G_{ij,x}^{op} s_i^{op} \right]$$

$$u_j, y = \sum_{i=1}^{N} \left[G_{ij,y}^{ss} s_i^{ss} + G_{ij,y}^{ds} s_i^{ds} + G_{ij,y}^{op} s_i^{op} \right]$$

$$u_j, z = \sum_{i=1}^{N} \left[G_{ij,z}^{ss} s_i^{ss} + G_{ij,z}^{ds} s_i^{ds} + G_{ij,z}^{op} s_i^{op} \right]$$

Green's functions

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- What kind of problem are we headed towards?

Green's Functions

- Analytical solution for elastic half-space exist
  - widely used formulation: *Okada, Y., Internal deformation due to shear and tensile faults in a half-space, Bull. Seismo. Soc. Amer., v. 82, 1018-1040, 1992.*
  - Original Fortran code is most reliable, implementations in other languages exist
- expressions for more complex earth structure exist
  - layered elastic
  - visco-elastic half space
  - elastic over visco-elastic

Solving for Slip

- Displacement at a point j on Earth's surface caused by slip on N fault patches can be written as:

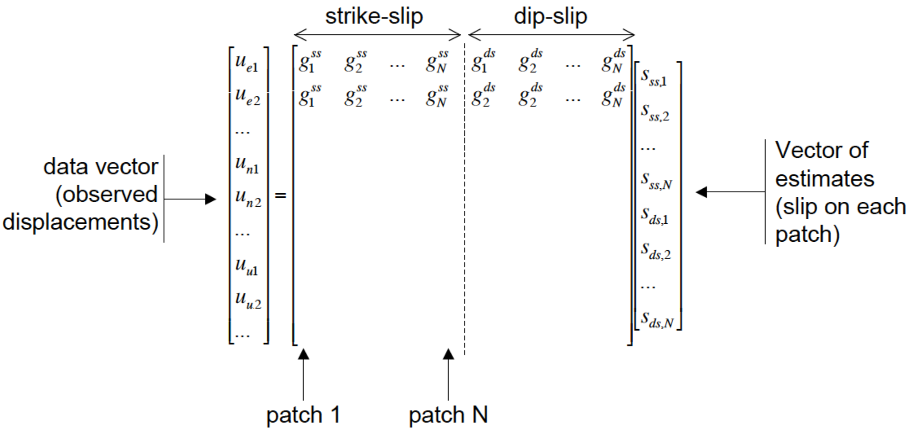
$$u_j = \sum_{i=1}^{N} G_{ij} s_i$$

- This looks familiar

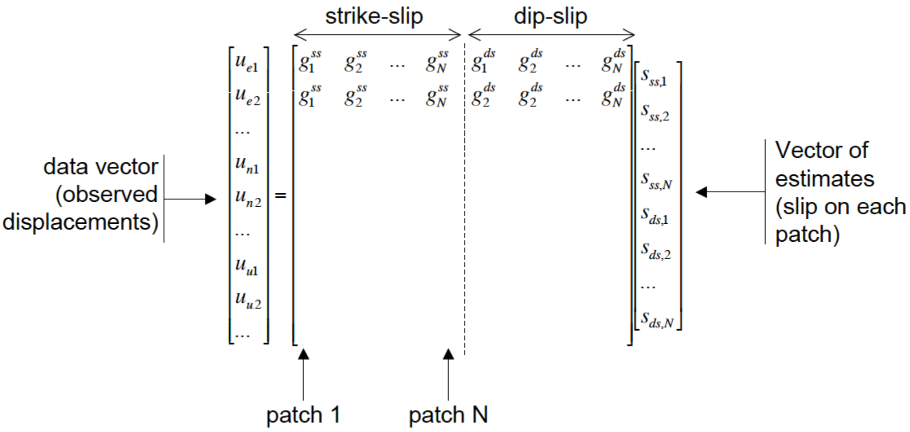
$$u = Gs$$

  - u is data vector
  - s is model vector
  - G is design matrix made of Green's functions

Solving for Slip



Solving for Slip



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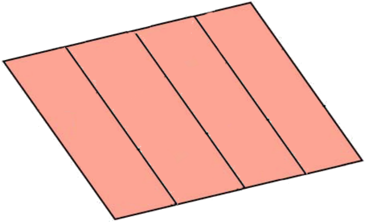
For prior 1D problems *G* was a matrix
How to deal with 2D problem of slip on fault?

Solving for Slip

This should be straight-forward to turn into G



Solving for Slip

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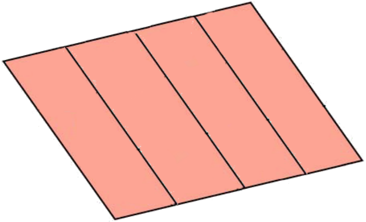
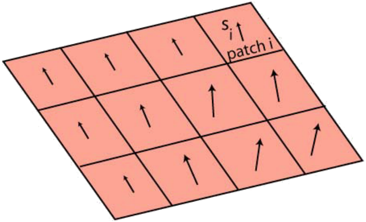


How about this?



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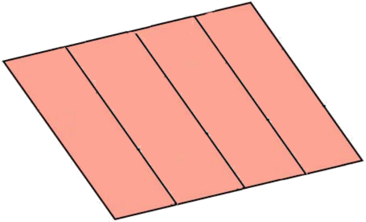

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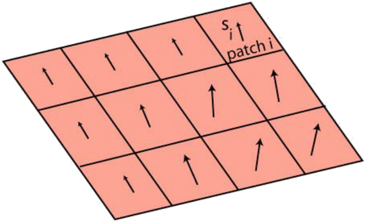
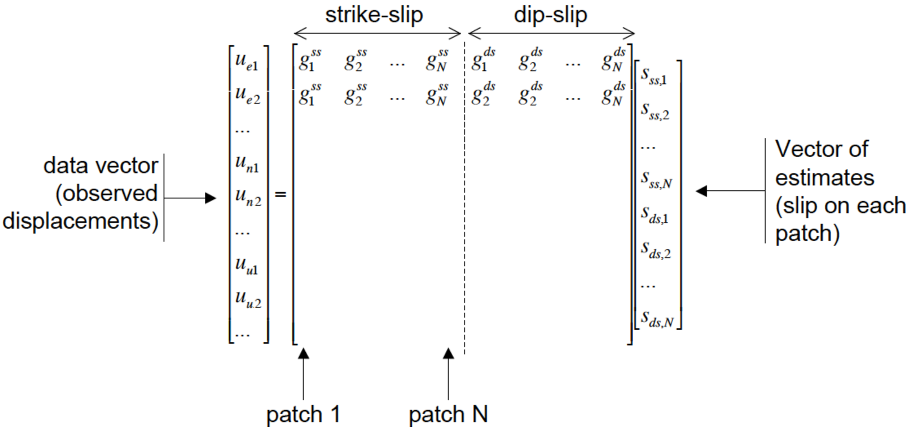


Linearize!

Solving for Slip



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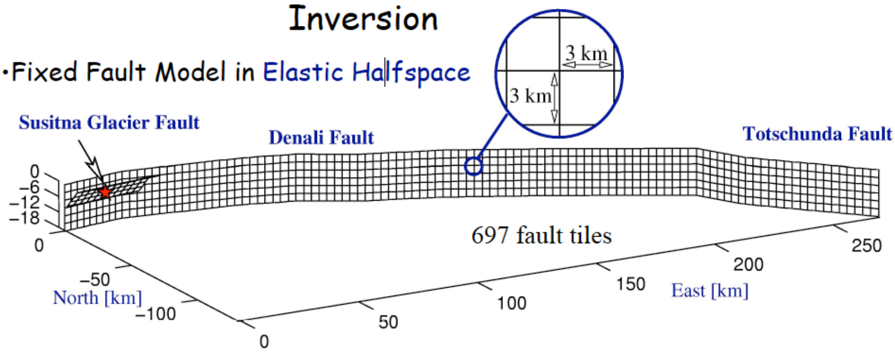
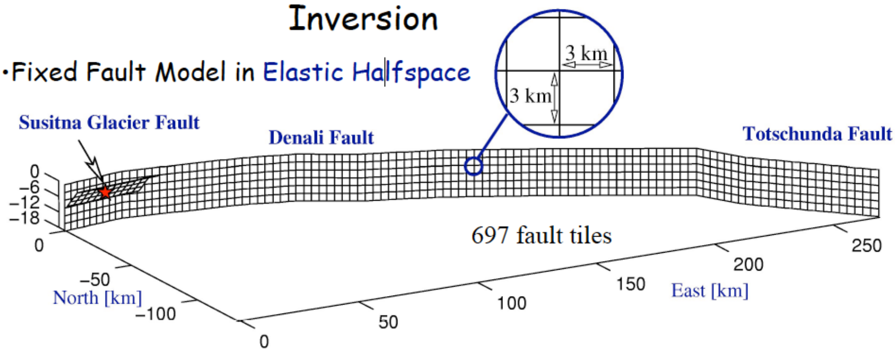
Solving for Slip



Sigrun Hreinsdottir

With 224 GPS sites and 697 fault tiles solving for dip-slip and strike-slip, what problem are we running into?

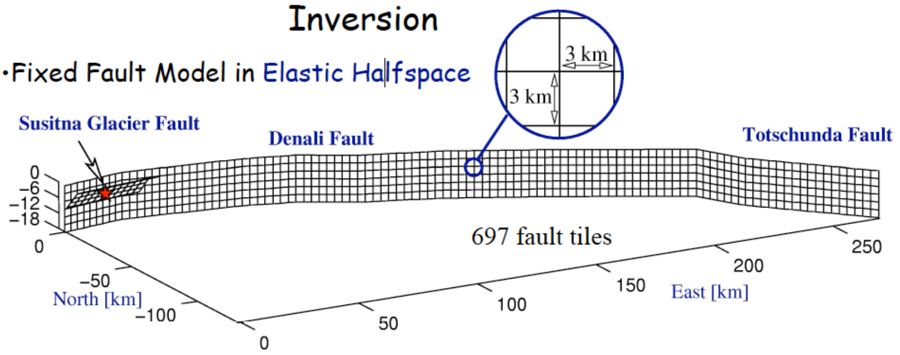
Solving for Slip



Sigrun Hreinsdottir

With 224 GPS sites and 697 fault tiles solving for dip-slip and strike-slip, what problem are we running into?
Underdetermined system.

Solving for Slip



Sigrun Hreinsdottir

- observations at 225 GPS sites: 675 data (if vertical helps)
- 697 fault tiles, ss, ds: 1394 unknowns
- no enough data to constrain number of unknowns
- also often an issue: unphysical oscillatory slip

Regularization / Smoothing

- Idea: Minimize the rate of change of slip with position
- "rate of change of slip" is curvature
- Laplacian:

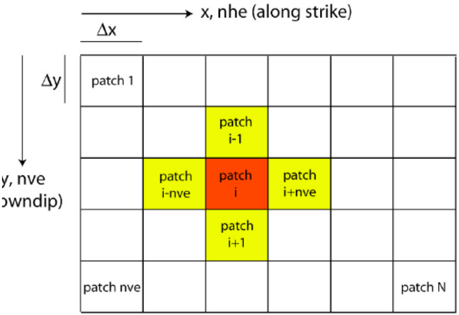
$$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$$

- Practice: Minimize sum of partial second differentials of slip for each fault patch
- Can be solved using finite-difference method for a function P

$$\frac{\delta^2 P(x)}{\delta x^2} \approx \frac{P(x - \Delta x) - 2P(x) + P(x - \Delta x)}{\Delta x^2}$$

Regularization / Smoothing

- Our function $P(x)$ is slip s which varies along-strike (x) and down-dip (y)
- For patch i finite difference approximation of Laplacian is (nve = number of vertical elements, nhe = horiztonal):



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$$l_i = \frac{s_{i-nve} - 2s_i + s_{i+nve}}{\Delta x^2} + \frac{s_{i-1} - 2s_i + s_{i+1}}{\Delta x^2}$$

Regularization / Smoothing

- In practice, equation:

$$l_i = \frac{s_{i-nve} - 2s_i + s_{i+nve}}{\Delta x^2} + \frac{s_{i-1} - 2s_i + s_{i+1}}{\Delta y^2}$$

is written in matrix form, for the along-strike and down-sip components:

$$\begin{bmatrix} \dots \\ 0 \\ \dots \end{bmatrix} = \underbrace{\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (0) & \frac{1}{\Delta x^2} & (0) & -\frac{2}{\Delta x^2} & (0) & -\frac{1}{\Delta x^2} & (0) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}}_{L_x} \begin{bmatrix} \dots \\ s_{i-nve} \\ \dots \\ s_i \\ \dots \\ s_{i+nve} \\ \dots \end{bmatrix} \qquad \begin{bmatrix} \dots \\ 0 \\ \dots \end{bmatrix} = \underbrace{\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (0) & \frac{1}{\Delta y^2} & (0) & -\frac{2}{\Delta y^2} & (0) & -\frac{1}{\Delta y^2} & (0) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}}_{L_y} \begin{bmatrix} \dots \\ s_{i-1} \\ \dots \\ s_i \\ \dots \\ s_{i+1} \\ \dots \end{bmatrix}$$

- The 2 Laplacian matrices are then added:

$$L = L_x + L_y$$

Regularization / Smoothing

- The original problem was:

$$[u] = \begin{bmatrix} G_{ss} & G_{ds} \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

- now it becomes:

$$\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{ss} & G_{ds} \\ L & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

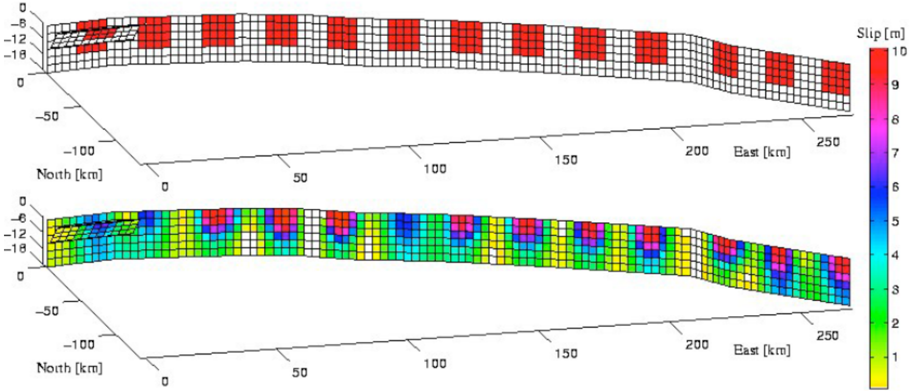
- amount of smoothing can be tuned using scalar smoothing factor κ:

$$\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{ss} & G_{ds} \\ \kappa L & 0 \\ 0 & \kappa L \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

- $\kappa = 0$: no smoothing, $\kappa = 1$ maximum smoothing

Regularization / Smoothing

What can you recover? Checker board / Resolution test:



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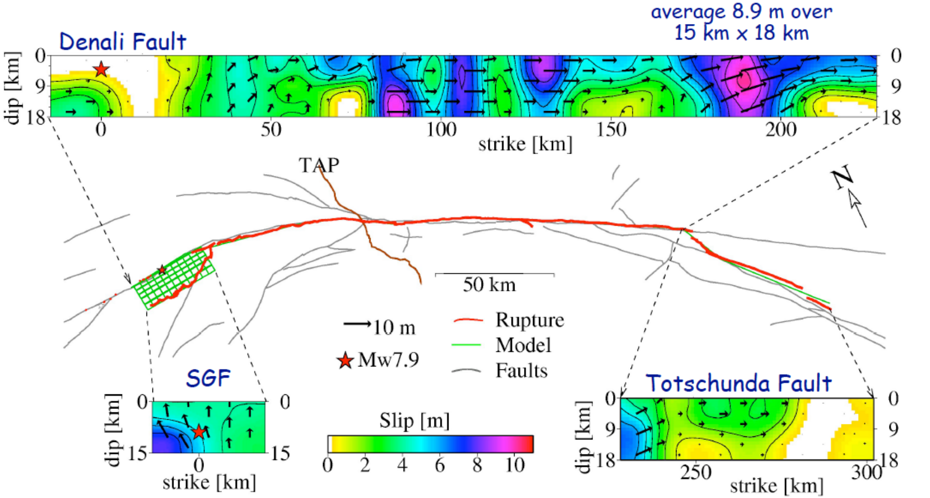
Distributed Slip Inversion

This is how you get this:



M_0=6.81x10^{20}Nm
M_w7.89

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